Asymptotic Reduction and Stirling's Formula

A Note on Partial Reduction in Analytic Structure

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Abstract

Analytic-to-algebraic reduction describes when infinite invariant structure admits a finite constraint. However, not all analytic structures reduce fully to algebraic form. In this note, we examine Stirling's formula as a canonical example of *asymptotic reduction*, in which an analytic invariant collapses not to a finite algebraic relation, but to a dominant asymptotic form. We show that factorial growth arises as a multiplicative aggregation invariant, and that Stirling's approximation extracts its leading-order behavior via analytic continuation and integral representation. This provides a concrete example of partial reduction within the framework of invariant formation, selection, and reduction.

1 Introduction

Analytic structures arise through infinite processes such as iteration, aggregation, and limits. In some cases, these structures admit exact algebraic reduction. In others, no such finite closure exists.

Between these extremes lies an intermediate case:

analytic structures that admit reduction only to dominant asymptotic form.

Stirling's formula provides a canonical example of this behavior.

2 Factorial as Aggregation Invariant

The factorial function is defined by:

$$n! = \prod_{k=1}^{n} k.$$

Taking logarithms yields:

$$\log n! = \sum_{k=1}^{n} \log k.$$

Thus factorial growth arises as an additive aggregation of contributions across discrete modes. Within the structural framework (Σ, A, Φ, I, P):

- Σ consists of integer-indexed configurations,
- Φ corresponds to multiplicative accumulation,

- I captures global growth behavior,
- P maps to analytic representation via logarithms.

3 Analytic Extension

The factorial extends analytically via the Gamma function:

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} \, dt.$$

This representation replaces discrete aggregation with a continuous integral weighted by exponential decay.

Thus:

- discrete sum $\sum \log k$ becomes an integral,
- multiplicative structure becomes exponential form,
- aggregation becomes a kernel-like integral.

4 Stirling's Approximation

Stirling's formula gives:

$$n! \sim \sqrt{2\pi n} \left(\frac{n}{e}\right)^n.$$

Equivalently:

$$\log n! \sim n \log n - n + \frac{1}{2} \log(2\pi n).$$

This expression captures the leading-order growth of the factorial.

5 Asymptotic Reduction

Unlike fixed-point or symmetry-based reduction, Stirling's formula does not produce a finite constraint of the form:

$$F(n!) = 0.$$

Instead, it produces an asymptotic invariant:

$$n! \approx \text{dominant growth form}.$$

An analytic invariant exhibits **asymptotic reduction** if it admits a simplified dominant form under infinite scaling, without reducing to a finite algebraic constraint.

6 Structural Interpretation

Stirling's formula may be interpreted as follows:

- The factorial encodes a global aggregation invariant.
- Analytic continuation (via Γ) transforms discrete aggregation into integral form.
- Asymptotic analysis extracts the dominant contribution to this integral.

In this sense:

$$\boxed{\text{Stirling's formula extracts the dominant invariant of a multiplicative aggregation process.}}$$

7 Relation to Reduction Framework

We distinguish three cases:

Type	Behavior
Full reduction	infinite $\to$ finite algebraic constraint
Asymptotic reduction	infinite $\to$ dominant growth form
No reduction	irreducible infinite structure

Stirling's formula lies in the second category.

8 Kernel and Spectral Perspective

The Gamma function representation:

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\, dt$$

can be viewed as an aggregation over weighted contributions.

Asymptotic evaluation (e.g., via saddle-point methods) identifies the dominant contribution to this integral.

Thus:

$$\boxed{\text{Asymptotic reduction corresponds to selecting the dominant contribution in a kernel-like aggregation.}}$$

9 Conclusion

Stirling's formula provides a concrete example of partial reduction in analytic structure. While it does not yield a finite algebraic constraint, it extracts the dominant invariant governing growth.

Not all analytic structure reduces to finite form; some reduces only to its leading invariant behavior.

This example refines the reduction framework by identifying asymptotic reduction as an intermediate class between full reduction and irreducible infinite structure.